DEPARTMENT OF MATHEMATICS HONOURS

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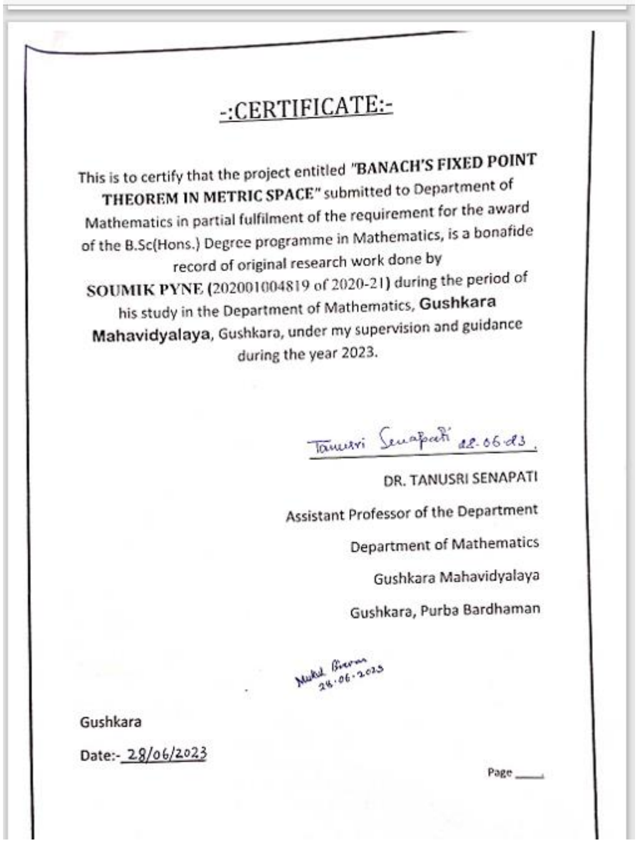
TITLE OF THE PROJECT:

BANACH'S FIXED POINT THEOREM IN METRIC SPACES

DURATION WITH DATE:

FEBRUARY, 2023 TO JUNE, 2023

PROJECT WORK COMPLETION CERTIFICATE



-:CERTIFICATE:-

This is to certify that the project entitled "**BANACH'S FIXED POINT THEOREM IN METRIC SPACE**" submitted to Department of Mathematics in partial fulfilment of the requirement for the award of the B.Sc(Hons.) Degree programme in Mathematics, is a bonafide record of original research work done by **SOUMIK PYNE** (202001004819 of 2020-21) during the period of his study in the Department of Mathematics, **Gushkara Mahavidyalaya**, Gushkara, under my supervision and guidance during the year 2023.

28.06.23

DR. TANUSRI SENAPATI

Assistant Professor of the Department

Department of Mathematics

Gushkara Mahavidyalaya

Gushkara, Purba Bardhaman

28.06.2023

Gushkara

Date:- 28/06/2023

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REPORT OF THE FIELD WORK: (PDF OF THE REPORT OF THE STUDENT) (PDF OF SOUMIK PYNE)

SAMPLE PHOTOGRAPH OF THE FIELD WORK:

PERMISSION LETTER FOR FIELD WORK FROM COMPETENT AUTHORITY

ACCORDING TO B.SC. HONS. SEM- 6 (DSE-4) SYLLABUS OF BURDWAN UNIVERSITY.

Unit-3 : Constraints and their classifications, Lagrange equation of motion for holonomic system, Gibbs-Appell's principle of least constraint, Work energy relation for constraint forces of shielding friction. 20L.

Course : BMH6PW01

Project Work (Marks : 75)

Any student may choose Project Work in place of one Discipline Specific Elective (DSE) paper of Semester -VI. Project Work will be done considering any topic on Mathematics and its Applications. The marks distribution of the Project work is 40 Marks for written submission, 20 Marks for Seminar presentation and 15 Marks for Viva-Voce.

Unit-3 : Constraints and their classifications, Lagrange's equation of motion for holonomic system, Gibbs-Appell's principle of least constraint. Work energy relation for constraint forces of shielding friction. 20L

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BANACH'S FIXED POINT THEOREM IN METRIC SPACES

DEPARTMENT OF MATHEMATICS

A Project report submitted to

GUSHKARA MAHAVIDYALAYA

For partial fulfilment of the requirement of

The B.Sc. (Hons.) Degree in Mathematics

By

SOUMIK PYNE

Reg No.: **202001004819 of 2020-21**

Roll No. **200311000040**

Under the supervision of

DR. TANUSRI SENAPATI

Department of Mathematics

Gushkara Mahavidyalaya

-:CERTIFICATE:-

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Tanusri Senapati 28.06.23

DR. TANUSRI SENAPATI

Assistant Professor of the Department

Department of Mathematics

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Gushkara, Purba Bardhaman

Mukul Biswas
28.06.2023

Gushkara

Date:- 28/06/2023

-:DECLARATION:-

I hereby declare that the project work entitled "BANACH'S FIXED POINT THEOREM IN METRIC SPACE" submitted to Gushkara Mahavidyalaya, Gushkara in partial fulfilment of the requirement for the award of Bachelor Degree of Science in Mathematics is a record of original project work done by me during the period of my study in the Department of Mathematics, Gushkara Mahavidyalaya, Gushkara

Soumik Pyne

SOUMIK PYNE

Gushkara

Date:- 28/06/2023.

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Introduction

It is well known that in real (or complex) analysis, the two pivotal concepts are those of convergence of sequence and continuity of functions. There too it is essentially the notion of distance given by the absolute value like $|x-y|$, which plays the underlying single key role. For carrying out abstract analysis without the aid of any algebraic structure like field or vector space, it becomes necessary to develop suitable axiomatic definition to start with. The intended machinary for such a development was accomplished chiefly along two directions. It was Frechet who in 1996 came forward with the idea of metric spaces followed by a further abstraction in 1914 by Hausdörff who initiated the splendid theories of general theories.

In a metric space, our prime task is to introduce an abstract formulation of the notion of distance between two points of an arbitrary non-empty set. It will be nice and interesting to see how with such a little appliance, we can generalized and talk about most of the central concepts of real (or complex) analysis like open and closed sets, limit point and compactness of sets, convergence of sequence, continuity and uniform continuity of functions etc. And that too without the least botheration for support of any structures — algebraic or otherwise.

Definition of metric Space

Let X be a non-empty set and $d: X \times X \to \mathbb{R}$ be a function satisfying the following axioms–

i) $d(x,y) \geq 0$, $\forall x,y \in X$ & $d(x,y) = 0$ iff $x=y$ (Non-negativity)

ii) $d(x,y) = d(y,x)$ (symmetric axiom)

iii) $d(x,z) \leq d(x,y) + d(y,z)$; $\forall x,y,z \in X$ (Triangle Inequality)

Then d is called a metric on X and (X,d) forms a metric Space

Fixed Point

Let (X,d) be a metric space. Then a point $x_0 \in X$ is said to be a fixed point of $T:(X,d) \to (X,d)$ if $T(x_0) = x_0$

Example

1) Let $X = \mathbb{R}$. Define $T: X \to X$ by $T(x) = \frac{x}{2}$, $\forall x \in X$.
   Then 0 is the only fixed point of T.
2) Define $T: \mathbb{R} \to \mathbb{R}$ by $T(x) = x^3$, $\forall x \in \mathbb{R}$.
   Here $x = 0, 1, -1$ are the only fixed points of T.
3) Define $T: \mathbb{R} \to \mathbb{R}$ by $T(x) = x + \sin x$, $\forall x \in \mathbb{R}$.
   Here the fixed points of T are given by $x = n\pi$; $n = 0, \pm 1, \pm 2, \ldots$
4) Define $T: \mathbb{C} \to \mathbb{C}$ by $T(x) = -x^3$, $\forall x \in \mathbb{C}$.
   Then the fixed points of T are $x = 0, \pm i$.

Banach Fixed Point Theorem

Let (X,d) be a complete metric space and let $T:(X,d) \to (X,d)$ be a contraction mapping. Then T has a unique fixed point in X.

Proof: Let $x_0 \in X$ be any point of X.

Let $x_1 = T(x_0)$

$$x_2 = T(x_1) = T(T(x_0)) = T^2(x_0)$$

$$x_3 = T(x_2) = T(T^2(x_0)) = T^3(x_0)$$

$$\vdots$$

$$x_{n+1} = T(x_n) = T^{n+1}(x_0)$$

$$\vdots$$

Now $d(x_2, x_1) = d(T(x_1), T(x_0)) \leq \alpha\, d(x_1, x_0)$; $0 < \alpha < 1$

$$d(x_3, x_2) = d(T(x_2), T(x_1)) \leq \alpha\, d(x_2, x_1) \leq \alpha^2 d(x_1, x_0)$$

$$\vdots$$

$$d(x_{n+1}, x_n) = d(T(x_n), T(x_{n-1})) \leq \alpha^n d(x_1, x_0)$$

$$\vdots$$

Conclusion

This project is an approach to the study of Banach's Fixed point theorem in metric space and its application to different field of mathematics. Starting with preliminaries, moving to definition of the topic and its application. We have seen that Fixed Point Theory plays an important role in mathematics and also on different topics apart from mathematics.

The Banach theorem seems somewhat limited. It seems intuitively clear that any continuous function mapping the unit interval into itself has a fixed point. We hope that this work will be useful for functional analysis related to normed spaces and fixed point theory. Our results are generalizations of the corresponding known fixed point results in the setting of Banach spaces on its normed space. Then all expected results in this project will help to understand better solution of complicated theorem.

Reference

1) M.N Mukherjee, Elements of Metric Spaces; Academic Publishers

2) KREYSZIG, Introductory Functional Analysis with Applications; Wiley & Sons.

Tanusri Senapati
28.06.23